II International Conference on Approximation Theory and Applications

Gaeta (Italy) • 14-18 September 2026

BOOK OF ABSTRACTS

II International Conference on

Approximation Theory and Applications

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Monday, September 14

15:00 – 16:00	**Registration**
16:00 – 16:10	**Opening Remarks**
	Chair: Gianluca Vinti
16:10 – 17:00	**Plenary Lecture: Wolfgang Dahmen** *Sparsity Notions and Model Classes for High-Dimensional Approximation*
17:00 – 17:30	**Coffee Break**
	Chair: Milvia Rossini
17:30 – 17:55	Len Bos *A Characterization of Optimal Prediction Measures via ℓ_1 Minimization*
17:55 – 18:20	Peter Binev *Adaptive Approximation of High Dimensional Data*
18:20 – 18:45	Raffaele Farina *Spectral Conditioning of Gram Matrices in Energy-Pooled Convolutional Classification Models*
19:30	**Welcome Cocktail and Dinner**

Tuesday, September 15

	Chair: Stefano De Marchi
09:00 – 09:50	**Plenary Lecture: Demetrio Labate** *Manifold learning via Fréchet maps*
09:50 – 10:15	Yurii Kolomoitsev *Sharp error estimates for Lagrange interpolation and Kantorovich-type operators in weighted L^p-spaces*
10:15 – 10:40	Rumen Uluchev *Higher order approximation of continuous functions by the Goodman-*

	Sharma-type operators
10:40 – 11:10	**Coffee Break**
	Chair: Laura Angeloni
11:10 – 11:35	Geno Nikolov *An inequality for Jacobi polynomials: a complement to Finite Increment Theorem*
11:35 – 12:00	Mirella Cappelletti Montano *On some classes of Gamma-type operators*
12:00 – 12:25	Flavia Lanzara *Fast computation of multi-dimensional volume potentials*
12:25 – 12:50	Domenico Fazzino *Regularized spline approximation of planar offset curves*
13:00	**Lunch Break**
14:00 – 16:00	**Free Time**
	Chair: Vita Leonessa
16:00 – 16:25	Alberto Cialdea *Completeness Theorems on the Boundary*
16:25 – 16:50	Domenico Vitulano *A High-Dimensional Multifractal FokkerPlanck Model for Interacting Financial Markets: Classical and Quantum Perspectives*
16:50 – 17:15	Michele Piconi *Simultaneous modular and norm approximation in Sobolev-Orlicz Spaces via semi-discrete operators*
17:15 – 17:45	**Coffee Break**
	Chair: Mirella Cappelletti Montano
17:45 – 18:10	Alfio Borzi *Finite-Block Volterra Structure and Approximation Properties in Reservoir Computing*
18:10 – 18:35	Nadaniela Egidi *Approximation of noisy scattered data by using RBF with different shape parameters*
18:35 – 19:00	Ilina Mihai *Evolution equations associated with positive linear operators*
20:00	**Dinner**

Wednesday, September 16

	Chair: Salvatore Cuomo
09:00 – 09:50	**Plenary Lecture: Gitta Kutyniok**
09:50 – 10:15	Wolfgang Erb *Max-Min Sampling Operators in Mathematical Morphology*
10:15 – 10:40	Milvia Rossini *Variably Scaled Kernels: A Kriging Perspective*
10:40 – 11:10	**Coffee Break**
	Chair: Alberto Cialdea
11:10 – 11:35	Danilo Costarelli *Embedding and characterization theorems for Lipschitz-type classes in Orlicz spaces generated by weak and strong moduli of smoothness*
11:35 – 12:00	Margherita Lampani *Physics-Informed Feature Maps for Kernel-Based Approximation*
12:00 – 12:25	Sergio Diaz-Elbal *Advances on probability density estimation with CS-RBFs: Towards anisotropic models*
12:25 – 12:50	Ilaria Arteconi *Interpolating Planar Point Data with Open and Closed Quintic PH B-Spline Curves*
13:00	**Lunch Break**
14:00 – 16:00	**Free Time**
	Chair: Michele Campiti
16:00 – 16:30	**Prize Session:** Rosario Corso *Beyond Linear Averaging: Nonlinear Operators with Power Averages*
16:30 – 17:00	**Prize Session:** Francesco Marchetti *Shaping kernels for function approximation*
17:00 – 17:30	**Prize Session:** Arianna Travaglini *Between Theory and Applications: Sampling-type Operators and Probabilistic Integral Operators in Function Spaces*
17:30 – 17:45	**Award Ceremony**
17:45 – 18:15	**Coffee Break**
18:15 – 19:00	**Poster Session**
20:00	**Conference Dinner (*)**

Thursday, September 17

	Chair: Elisa Francomano
09:00 – 09:50	**Plenary Lecture: Ana Maria Acu** *Approximation, shape preservation and image reconstruction by neural network operators*
09:50 – 10:15	Rosanna Campagna *Adaptive Generalized P-Splines for Functional Data Analysis: Approximation, Prediction, and Uncertainty Quantification*
10:15 – 10:40	Lorella Fatone *A nonlinear diffusion method for image processing*
10:40 – 11:10	Lucrezia Rinelli *Shape-preserving properties and localization results for max-product sampling Kantorovich operators*
11:10 – 11:30	**Coffee Break**
11:30 – 12:50	**General Assembly of the UMI – T.A.A. Group**
13:00	**Lunch Break**
14:00 – 20:00	**Free Time**
20:00	**Dinner**

Friday, September 18

	Chair: Mariantonia Cotronei
09:00 – 9:25	Tomasz Beberok *Spectral Operator and L^2 Bernstein Inequalities on the Ball*
09:25 – 9:50	Silvia Marconi *Learning Under Environmental Shift: An Approximation-Theoretic Analysis of Climate Prediction*
09:50 – 10:15	Georgian-Cristian Chivu *Deep Architectures for Contrast-Free Aortic Lumen Segmentation: A Systematic Benchmark*
10:15 – 10:40	Alessandro Bottino *Guided Physics-Informed Neural Networks for an Inverse Elliptic Flow Problem with Dirac-Type Source in Randomly Heterogeneous Porous Media*
10:40 – 11:10	**Coffee Break**

	Chair: Stefano De Marchi
11:10 – 11:35	Maryam Mohammadi (online) *CS-RBF Test Spaces for Patchless Meshfree VPINNs*
11:35 – 12:00	Mojtaba Ranjbar (online) *Spectral Kernel Design in RKHS: A Tunable Meshless Collocation for Neutral Delay Equations*
12:00	**Closing Remarks**
13:00	**Lunch**

(*) During the Conference Dinner, the winner of the Best Poster Award sponsored by De Gruyter Brill will be announced

PLENARY TALKS

II International Conference
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Approximation, shape preservation and image reconstruction by neural network operators

Ana Maria Acu
Department of Mathematics and Informatics, Lucian Blaga University of Sibiu

Neural Network (NN) operators have emerged as a powerful bridge between classical approximation theory and modern data-driven methodologies. In this talk, we present several recent developments concerning NN operators generated by sigmoidal and B-spline activation functions. We discuss their approximation behavior, shape-preserving properties, and asymptotic analysis, including Korovkin-type results and Voronovskaja formulas that describe the local behavior of the approximation process. Special attention is devoted to neural network operators based on central B-splines, whose compact support and smoothness lead to localized and stable approximation schemes. Their multivariate extensions are also considered, together with preservation properties and convergence results in the bivariate setting. The theoretical developments are complemented by applications to image processing. In particular, we analyze the performance of Neural Network and Sampling Kantorovich operators in image reconstruction and denoising problems affected by Gaussian, Poisson, and Speckle noise. Numerical experiments, supported by standard image quality measures, show that operator-based approaches provide accurate and robust reconstructions, highlighting the potential of approximation-theoretic methods in modern imaging applications.

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Sparsity Notions and Model Classes for High-Dimensional Approximation

Wolfgang Dahmen
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Approximating, in particular, learning operators acting between function spaces, plays a central role in Scientific Machine Learning. Its main motivation is to facilitate an efficient evaluation of the mapping that takes a given class of "problem data" such as source terms, boundary data, or uncertain coefficient fields into the corresponding solution of a system of corresponding partial differential equations (PDEs). I On the one hand, a learned operator surrogate is to facilitate the exploration of corresponding solution manifolds via such efficient forward simulations. On the other hand, such operator surrogates are to help solving inverse problems such as recovering, from a finite number of observational data or measurements, a particular state in the solution manifold, or even the unknown problem data behind that state. In this talk we highlight some essential obstructions, namely the intrinsic high dimensionality of such recovery tasks as well as their ill-posedness to discuss then - in a possibly non-technical manner- concepts that can overcome or mitigate these obstructions. This concerns the interplay between high dimensional sparsity notions, statistical estimation, and ways how to best link physics information, represented by the PDE model, with data information through the use of proper model compliant metrics.

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Expressivity, Stability, and Sustainability of Spiking Neural Networks

Gitta Kutyniok
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Spiking neural networks (SNNs) are increasingly viewed as a promising foundation for next-generation artificial intelligence due to their event-driven computation, temporal processing capabilities, and potential for highly energy-efficient implementations on neuromorphic hardware. In contrast to the current trend of continuously scaling classical deep learning architectures, SNNs offer a biologically inspired and potentially more sustainable alternative for temporal and sequential learning tasks.

In this talk, we discuss recent advances on the mathematical foundations of recurrent spiking neural networks with a focus on expressivity, dynamics, and stability. We first study the approximation power of discrete-time and recurrent SNN architectures for sequence modeling and analyze under which conditions spike-based systems can efficiently encode complex temporal dependencies. We then investigate random spiking neural networks and show that they exhibit remarkable stability properties together with a surprisingly simple spectral structure, providing new insights into their trainability and dynamical behavior.

These results contribute to a deeper theoretical understanding of spike-based computation and highlight how mathematically principled SNN architectures may enable reliable and sustainable AI systems beyond the paradigm of pure scale.

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Manifold learning via Fréchet maps

Demetrio Labate
Department of Mathematics, University of Houston

This work is motivated by neuroimaging applications, where correlation matrices derived from functional Magnetic Resonance Imaging (fMRI) data model neural connection strengths to evaluate brain function. Because these matrices are typically high-dimensional (e.g., 2000 x 2000), they impose a significant computational burden on downstream clustering and classification tasks. Current approaches often address this challenge either by disregarding the underlying non-Euclidean geometry or by employing rank-reduction techniques that incur significant information loss. To bridge this gap, we introduce a novel framework for dimensionality reduction on a large class of Riemannian manifolds, including the space of correlation matrices. The core innovation of our approach is the application of Fréchet maps - a class of nonlinear transformations tailored to achieve substantial dimensionality reduction while preserving the intrinsic geometry of the data. In this presentation, I will detail the mathematical foundations of this framework, address parameter optimization, and demonstrate its performance through numerical experiments. This is joint work with Robert Azencott, Nicolas Charon, Andreas Mang, and Ji Shi.

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UMI-TAA PRIZE TALKS

Beyond Linear Averaging: Nonlinear Operators with Power Averages

Mirella Cappelletti Montano
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Rosario Corso[1]
Dipartimento di Matematica e Informatica, Università degli Studi di Palermo
Vita Leonessa
Dipartimento di Ingegneria, Università degli Studi della Basilicata

Kantorovich-type operators are among the most widely studied linear operators in Approximation Theory. Their purpose is to approximate functions through linear averages of the target function taken over appropriate subsets of the domain.

Although linear averages are widely used in mathematics, power averages also play a significant role. Motivated by this reason and by the properties of power averages, a recent development in the theory of Kantorovich-type operators was presented in [1] through the introduction of nonlinear operators defined by means of power averages of the function under approximation.

The definition from [1] is the following. Let $p > 0$ and $n \geq 1$. The *Kantorovich operator with p-averages $K_{p,n}$* is defined by

$$(K_{p,n}f)(x) = \sum_{k=0}^{n} \left((n+1) \int_{\frac{k}{n+1}}^{\frac{k+1}{n+1}} f(t)^p \mathrm{d}t \right)^{\frac{1}{p}} B_{k,n}(x), \qquad x \in [0,1],$$

for $f \in L^p(0,1)$, $f \geq 0$, and by

$$K_{p,n}f = K_{p,n}(f_+) - K_{p,n}(f_-),$$

for all $f \in L^p(0,1)$, where $\{B_{k,n} : n \geq 1, 0 \leq k \leq n\}$,

$$B_{k,n}(x) = \binom{n}{k} x^k (1-x)^{n-k}, \quad 0 \leq k \leq n,\, x \in [0,1],$$

is the Bernstein basis and f_+, f_- are the non-negative and non-positive parts of f, respectively. For $p = 1$ the classical Kantorovich operator is recovered. In contrast, for $p \neq 1$ the operator $K_{p,n}$ is nonlinear, and this makes the study much more challenging than the linear case.

In this talk, we discuss the motivating ideas, the properties of the operators $K_{p,n}$, and their convergence results, together with a brief outlook on future developments. Particular emphasis is placed on one of the main result obtained in [1], namely the Voronovskaya-type asymptotic formula, that is an explicit expression of

$$\lim_{n \to +\infty} n((K_{p,n}f)(x) - f(x))$$

when f is twice differentiable at x.

References

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[1]This work was partially supported by INdAM - GNAMPA Project 2026 "Problemi di approssimazione tramite operatori non lineari, stime ottimali e spazi frazionari".
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Shaping kernels for function approximation

Francesco Marchetti
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Kernel-based methods provide a versatile framework for function approximation, which is well grounded in the theory of Reproducing Kernel Hilbert Spaces (RKHSs). However, their effectiveness depends critically on the choice of the kernel and on how well it reflects the structure of the underlying problem.

In this talk, we discuss both less and more recent ideas for *shaping kernels*, namely modifying kernel models to improve approximation accuracy and incorporate prior information. Starting from strategies for shape-parameter selection, we then consider mapped kernels, including variably scaled kernels, which adapt the geometry induced by the kernel through scaling functions or domain transformations. These techniques enable the approximation process to better capture localized features, anisotropies, and complex structures in the target function. We also discuss interactions with statistics and machine learning, highlighting how data-driven techniques can be employed to design and adapt kernels, as well as how the approximation results presented here can be interpreted in a statistical framework.

Alongside the theoretical foundation, numerical results will show the benefits of these approaches in approximation problems.

References

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Between Theory and Applications: Sampling-type Operators and Probabilistic Integral Operators in Function Spaces

Arianna Travaglini
Department of Physics and Geology, University of Perugia, Italy

This talk presents two main lines of research, developed at both the theoretical and applicative levels, and connected by a common thread. The first line concerns the study of sampling-type operators [1]. We establish pointwise, uniform, and modular convergence in Orlicz spaces for nonlinear sampling Durrmeyer operators [2], providing a unified framework encompassing L^p spaces and including nonlinear extensions of generalized and Kantorovich sampling operators as special cases; the multivariate setting is also addressed [3]. Applications of the sampling Kantorovich (SK) algorithm to several biomedical imaging tasks [4–7], such as volumetric estimation of Alzheimer's biomarkers from MRI, retinal OCTA image analysis for diabetic retinopathy, and aortic lumen extraction from CT scans, are also discussed. Comparisons between SK and Neural Network operators with several denoising methods confirm accurate and stable reconstructions across noise models and imaging modalities [8]. The second line concerns the study of operators constructed via probability distributions. Multiparameter distributions offer greater flexibility for large empirical datasets, with applications to software quality assessment and information-theoretic modeling. We study operators based on the generalized Gamma and power Lindley distributions, proving convergence in weighted continuous function spaces and L^p spaces as well as asymptotic formulas [9, 10]. The multidimensional extension of the generalized Gamma-type operators and applications to image denoising are also presented [11].

References

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CONTRIBUTED TALKS

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Interpolating Planar Point Data with Open and Closed Quintic PH B-Spline Curves

Ilaria Arteconi[1], *Martina Balla*, *Carolina Vittoria Beccari*[1], *Lucia Romani*[1]
[1]Department of Mathematics, University of Bologna

Pythagorean hodograph curves are parametric polynomial curves whose first derivative has a Euclidean norm that is itself a polynomial. This distinctive property makes them especially useful whenever the accurate computation of arc length and offset curves (namely curves defined at a prescribed distance along the normal direction) is required, as in robotics, motion control, and collision detection.

Since B-splines are the standard representation in CAD/CAM systems, research has increasingly focused on PH curves expressed in the B-spline basis, known as PH B-spline curves. These curves were introduced in [1,2] and have since been the subject of several recent studies.

In this presentation, we address the problem of interpolating a sequence of planar points with an open or closed quintic PH B-spline curve. By exploiting the curve representation proposed in [3], we formulate the interpolation problem as a system of nonlinear equations, which can be efficiently solved by the Newton–Raphson method using a suitable choice of the initial point. Numerical examples will be presented to illustrate how the proposed method produces visually pleasing curves, even for datasets describing complex shapes that are typically challenging for interpolation.

References

[1] G. Albrecht, C. V. Beccari, J.-C. Canonne, L. Romani, Pythagorean-Hodograph B-Spline Curves, Computer Aided Geometric Design, 57 (2017), 57–77.

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Spectral Operator and L^2 Bernstein Inequalities on the Ball

Tomasz Beberok [1]
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Yuan Xu
Department of Mathematics, University of Oregon, Eugene, OR 97403–1222, USA

In this talk, I will present results from joint work with Yuan Xu, contained in the paper *"On Bernstein Inequalities on the Unit Ball"*. The paper establishes new Bernstein-type inequalities for algebraic polynomials on the unit ball. In particular, we study Bernstein inequalities in the L^2 norm on the unit ball. The proofs are based on the spectral operator, a second-order differential operator whose eigenfunctions are orthogonal polynomials.

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[1]The first author was supported by the Polish National Science Centre (NCN) Miniatura grant no. 2025/09/X/ST1/00082. The second author was partially supported by Simons Foundation Grant #849676.
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Adaptive Approximation of High Dimensional Data

Peter Binev [1]
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We consider adaptive approximation of the regression function corresponding to data points with unknown distribution in high dimensions. The case of lower intrinsic dimension of the point clouds is of particular interest. The domain is partitioned into simplices using a multidimensional variant of the newest vertex bisection on the plane. The adaptive approximation utilizes the idea of *sparse occupancy tries* [1] to avoid some curse-of-dimensionality issues. While the piecewise constant approximation is well understood, the use of local approximation by higher order polynomials is problematic due to the possible unbalanced distribution of the point cloud within the cells of the partition. To provide higher order approximation with high probability, we propose to use a distribution-based local approximation scheme that incorporates an estimation of the distribution of the local point cloud and finding a mapping transforming it to a nearly uniform distribution.

References

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[1]Supported in part by NSF DMS 2038080, and 2245097.
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Finite-Block Volterra Structure and Approximation Properties in Reservoir Computing

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Reservoir computing is investigated through the finite-block Volterra structure generated by finite-width reservoirs. For linear and quadratic logistic activations, the induced input-to-state map admits an exact finite Volterra representation, providing an explicit characterization of the associated hypothesis class. This viewpoint reveals that finite reservoir width imposes rank constraints on the admissible Volterra kernels and establishes direct connections with low-rank approximation.

In the quadratic case, the rank constraints reduce to classical matrix-rank limitations, leading to approximation estimates in terms of spectral decay and best low-rank approximation error. A Volterra-Barron complexity measure is introduced for finite-block polynomial functionals, and corresponding approximation rates are established. Numerical experiments illustrate the theoretical results and highlight the role of kernel rank and spectral structure in reservoir approximation.

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A Characterization of Optimal Prediction Measures via ℓ_1 Minimization

Len Bos
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Suppose that $K \subset \mathbb{C}$ is compact and that $z_0 \in \mathbb{C}\backslash K$ is an external point. An optimal prediction measure for regression by polynomials of degree at most n, is one for which the variance of the prediction at z_0 is as small as possible. Hoel and Levine ([2]) have considered the case of $K = [-1, 1]$ and $z_0 = x_0 \in \mathbb{R}\backslash[-1, 1]$, where they show that the support of the optimal measure is the $n+1$ extremme points of the Chebyshev polynomial $T_n(x)$ and characterizing the optimal weights in terms of absolute values of fundamental interpolating Lagrange polynomials. More recently, [1] has given the equivalence of the optimal prediction problem with that of finding polynomials of extremal growth. They also study in detail the case of $K = [-1, 1]$ and $z_0 = ia \in i\mathbb{R}$, purely imaginary. In this work we generalize the Hoel-Levine formula to the general case when the support of the optimal measure is a finite set and give a formula for the optimal weights in terms of an ℓ_1 minimization problem.

References

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Guided Physics-Informed Neural Networks for an Inverse Elliptic Flow Problem with Dirac-Type Source in Randomly Heterogeneous Porous Media

Salvatore Cuomo, Severino Gerardo, Vincenzo Vocca, Alessandro Bottino [1]
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We study an inverse groundwater flow problem with localized pumping/injection sources in randomly heterogeneous porous media. The problem is challenging because the governing PDE is singular, the medium is heterogeneous, and hydraulic-head data alone provide limited parameter identifiability. We introduce a nondimensional scaling based on the geometric mean conductivity, source strength, and horizontal correlation length. This reduces the inverse PINN task to estimating two physically relevant heterogeneity descriptors: the log-conductivity variance σ_Y^2 and the anisotropy ratio $\Theta = \ell_y/\ell_x$.

The method is tested on a 3×3 benchmark grid with $\sigma_Y^2 \in 0.1, 1, 10$ and $\Theta \in 0.5, 1, 2$, comparing a borehole-informed setting, with sparse observations of both hydraulic head and log-conductivity, against a head-only setting. When borehole data are available, both parameters are recovered with sub-percent errors. With head-only data, the method remains effective in the intermediate-variance regime, while weak and very strong heterogeneity expose clear identifiability limitations.

These results show that nondimensional scaling and careful parameter selection are essential in inverse PINNs: they remove non-identifiable degrees of freedom and focus the approximation problem on the heterogeneity features supported by the data.

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[1] The authors gratefully acknowledge the Institute for Sustainable Plant Protection, National Research Council of Italy (CNR-IPSP), Portici (NA). The authors also acknowledge the University of Naples Federico II for institutional support. A. Bottino, S. Cuomo and V. Vocca acknowledge GNCS-INdAM and the UMI-AI-ML, UMI-MIVA and UMI-TAA research groups.

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Adaptive Generalized P-Splines for Functional Data Analysis: Approximation, Prediction, and Uncertainty Quantification

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Functional data analysis (FDA) is a branch of statistics that goes beyond the analysis of individual data points and treats curves, surfaces, or images as individual observational units.

In this context, we integrate FDA methods with spline based numerical approximation. We propose an adaptive generalized P-spline, aiming to capture the nuances of evolutionary phenomena emerging in various scientific fields through the analysis, prediction, and quantification of the proposed model's uncertainty.

The classical penalized regression P-spline [1] is characterized by a finite difference second order penalty term and by equispaced knots. We present a novel approximation framework generalizing the P-splines introducing both a first order penalty term and a system of non-uniform and adaptively placed knots [2], inspired by the free knot spline paradigm [3]. The key innovation of our proposal is the integration of a conditioning-aware strategy for selecting both the number and positions of the knots, as well as the two regularization parameters, so controlling the model complexity while ensuring numerical stability.

Once predicting functional data, a significant challenge remains in quantifying the uncertainty of these predictions. We enrich our framework with a Conformal Prediction approach [4] to define regions of uncertainty for predictions.

Extensive numerical experiments conducted on both synthetic and real-world datasets demonstrate that the resulting approximation framework, not only improves the fit across the entire functional domain, significantly outperforming traditional free knots and smoothing spline methods, but also guarantees a global predictive coverage across the entire functional domain, converting deterministic functional predictions into statistically valid prediction bands.

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On some classes of Gamma-type operators

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The talk deals with two classes of integral operators of probabilistic type, introduced in [1] and [2] (see [3] for the multivariate case), which are constructed by means of the generalized Gamma distribution. We discuss their approximation properties in weighted continuous functions spaces, providing some estimates of the rate of convergence by means of different moduli of smoothness as well as an asymptotic formula. In particular, we discuss some applications to Semigroups Theory of one of the two classes of operators, which preserves linear functions and, in view of this property, is more suitable to such applications.

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Deep Architectures for Contrast-Free Aortic Lumen Segmentation: A Systematic Benchmark

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Segmenting the patent aortic lumen from *non-contrast* CT is a clinically relevant inverse problem: the iodinated contrast medium that reveals the blood-carrying vessel is nephrotoxic and contraindicated in many patients with abdominal aortic aneurysm. The task is to recover the lumen mask directly from the basal image I_B, where the boundary is faint and calcified plaque must be excluded.

Classical approaches rest on *sampling-type approximation operators*: a non-AI baseline built from Sampling–Kantorovich operators reconstructs the contrast field and thresholds it (Dice $\approx$0.75), while a U-Net trained with an SSIM loss to synthesise the contrast image I_{CM} from I_B reaches $\approx$0.81 [1]. In earlier work within our group, a comparison of seventeen encoder–decoder variants on the same 128×128 data set a stronger benchmark: a dual-attention U-Net at Dice 0.8937 and 95th-percentile Hausdorff distance HD95 $= 2.68\,\mathrm{mm}$.

Starting from this benchmark we introduce four contributions: (i) a *2.5D* multi-slice input giving longitudinal vessel context at no architectural cost; (ii) a *topology-aware* loss combining Dice with a differentiable radius-weighted centreline–boundary surrogate (a soft-erosion approximation of cbDice [2,3]) targeting HD95 and the aortic-dissection case, where a false lumen splits the vessel; (iii) a *multi-task* shared-encoder, dual-decoder network that jointly segments the lumen and synthesises I_{CM}, merging the previous two-network pipeline into one model and using contrast synthesis as free auxiliary supervision; and (iv) a benchmark of modern architectures – MedNeXt [4], a residual-attention U-Net, U-KAN [5], VM-UNet [6] and the self-configuring nnU-Net [7] – on a fixed 6-patient held-out test set with identical pre- and post-processing.

The residual-attention U-Net with 2.5D context currently reaches Dice 0.8725; the composite model (2.5D, topology loss, multi-task synthesis, 5-fold ensemble) is projected to exceed the 0.8937 benchmark on all metrics. We discuss the interplay between sampling-operator reconstruction and learned segmentation, and the role of topology-aware losses for tubular contrast-free vascular structures.

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Completeness Theorems on the Boundary

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Broadly speaking, there are two types of completeness theorem for systems of particular solutions of partial differential equations. The first type of result shows that, in a given norm, a solution to a partial differential equation in a domain can be approximated by a sequence of particular solutions to the same equation. For instance, if we have a holomorphic function f of one complex variable, we can ask under what conditions f can be approximated by polynomials or rational functions in certain norms. The classical theorems of Runge and Mergelyan are the main results in this area. These problems have been widely studied and extended to general elliptic partial differential equations. These results suggest using a sequence of particular solutions of the equation to approximate the solution of a given BVP for such an equation. However, they do not provide any information about convergence to the desired solution, even when a computational method is employed.

The results of the second type are known as completeness theorems in the sense of Picone. These are much more sophisticated and refer not only to a PDE, but also to a certain BVP. Given a BVP in a domain Ω, the aim is to prove that the system obtained by applying the BVP's boundary operator to specific PDE solutions is complete in an appropriate space on the boundary $\partial\Omega$. If this completeness holds, it is possible to find a sequence such that its boundary data sequence converges to the boundary data of the problem. As a consequence of results similar to the maximum principle, we obtain a sequence that converges to the solution of the BVP throughout Ω.

The aim of the present talk is to discuss this second topic. In particular, we will focus on the completeness on the boundary of systems obtained by means of polynomial solutions. After a review of the previous results, we present the most recent ones. They concern elasticity system [1], higher order scalar elliptic equations in two variables [2], parabolic equations [3,5] and the Moisil-Theodorescu system [4].

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Embedding and characterization theorems for Lipschitz-type classes in Orlicz spaces generated by weak and strong moduli of smoothness

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Two different notions of moduli of smoothness in Orlicz spaces generated by convex φ-functions have been considered ([1,2]): the first defined by means of the Luxemburg norm (strong moduli of smoothness) and the second by the usual integral modular functional of the space (weak moduli of smoothness). For these tools we establish the equivalence with corresponding strong and weak K-functionals based on Sobolev-Orlicz spaces. From the above definitions, two different versions of the Lipschitz and the generalized Lipschitz classes can be deduced: the strong and the weak ones. Embeddings, inclusions and characterization theorems have been proved between the strong and the weak versions of the above classes, with some classical and modified versions of usual functional spaces. For instance, embeddings between Lipschitz and generalized Lipschitz classes of the same order have been proved, as well as a sort of converse inclusions with specific choices of the considered parameters. The equivalence between the strong Lipschitz classes and the Sobolev-Orlicz spaces has been established when φ is an N-function satisfying certain growth conditions (namely the Δ_2 and the ∇_2 conditions, see [3,4]). When φ is not an N-function, an embedding result between the strong Lipschitz classes and spaces of bounded φ-variation functions can be established. Finally, in order to prove characterization theorems for the weak Lipschitz and generalized Lipschitz classes, a weak (modular) generalization of the Besov-Orlicz spaces is introduced. The latter have been proved to be interpolation spaces between the Sobolev-Orlicz and the Orlicz spaces.

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Advances on probability density estimation with CS-RBFs: Towards anisotropic models

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Compactly Supported Radial Basis Functions (CS-RBFs) are a cornerstone of multivariate approximation theory, yet their role in probabilistic modeling remains largely unexplored. This work formalizes the Wendland C^2 family $\phi(r) = (1-r)_+^4(4r+1)$ [1] as a parametric family of probability density functions. A univariate CS-RBF density on a domain $\Omega = [a, b]$ is defined as

$$f(x) = \frac{1}{N}\,\phi\big(\varepsilon|x-c|\big), \qquad x \in [a, b],$$

with center $c \in \mathbb{R}$, shape parameter $\varepsilon > 0$, and normalization constant N. Exploiting the polynomial structure of ϕ, and its compact support, we derive explicit expressions for the main distributional properties: normalization constants, moments of arbitrary order k, and the cumulative distribution function. When the natural support of the CS-RBF is truncated by the variable's domain Ω, a common problem in density estimation [2], the distribution properties can be computed exactly through polynomial primitives.

In the multivariate setting, a CS-RBF density takes the form

$$f(\mathbf{x}) = \frac{1}{N}\,\phi\big(\varepsilon\|\mathbf{x}-\mathbf{c}\|\big), \qquad \mathbf{x} \in \mathbb{R}^d,$$

where the normalization constant N is available in closed form in the untruncated case due to the radial nature of the function. Conditioning on known values $\mathbf{x}_{2:d}$ reduces this density to a univariate slice whose support can be analytically determined and is given by $\Delta = \big(\varepsilon^{-2} - \rho^2\big)^{1/2}$, where $\rho = \Big(\sum_{i=2}^{d}(x_i - c_i)^2\Big)^{1/2}$ is the distance from the conditioning point to the projected center. Explicit expressions for the normalization, CDF, and moments up to order $k = 2$ of the conditional density $f(x_1 \mid \mathbf{x}_{2:d})$ are obtained via hyperbolic primitives.

Density estimation is performed via a mixture of CS-RBFs, $p(\mathbf{x}) = \sum_j w_j f_j(\mathbf{x})$. This model inherits all the components' properties linearly, so mixture moments, marginals, and conditional CDFs follow directly from those of the individual components.

Two directions arise from this framework. The isotropic Euclidean norm can be replaced by a Mahalanobis norm (A-norm), $\|\mathbf{x}\|_A = \sqrt{\mathbf{x}^\top A\,\mathbf{x}}$, allowing the multivariate densities to capture correlations more effectively, but posing a challenge in conditioning due to the asymmetrical geometry of the CS-RBFs. Additionally, the compact support of the functions and the availability of explicit first and second moments for each component and its conditionals opens the way to moment-based EM fitting algorithms, offering an alternative to gradient-based optimization with potentially improved convergence and interpretability.

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Approximation of noisy scattered data by using RBF with different shape parameters

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Radial basis functions (RBFs) play an important role in interpolation and provide robust meshless methods for PDE solving, machine learning, artificial intelligence, with many implications in various fields of application. The associated linear system has a dense coefficient matrix, which means it takes quadratic time to assemble and cubic time to solve. To decrease the computational cost, various techniques have been developed, that use fast multipole methods or hierarchical matrices or wavelet matrix compressions.

Here, we consider scattered data approximation for multivariate data sets by globally supported RBFs, such as Gaussian and Inverse Multiquadric RBFs, whose shape parameter is crucial to their approximation performance. Indeed, this parameter governs the balance between accuracy and stability of the corresponding method, and usually optimization techniques are used to select an optimal shape parameter. We propose a method to compute the shape parameter based on the distribution of the data points near the corresponding RBF center. We test the proposed method with noisy data.

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Max-Min Sampling Operators in Mathematical Morphology

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In this talk, we build a conceptual bridge between mathematical morphology and max-min sampling operators. This bridge will allow us to obtain a deeper understanding of the interrelations between continuous and discrete morphological processes and new insights into the convergence behavior of discrete morphological transformations as the spatial resolution of the data increases. To this end, we introduce and analyze non-linear sampling operators based on the arithmetic operations of the max-min algebra. These new sampling operators can be interpreted as approximate morphological operations relying on the function values on a chosen sampling grid and a specific kernel. We investigate the convergence behavior of these operators, alongside with the respective properties of their morphological operations as dilation, erosion, opening and closing. In the two-dimensional setting, we show how these morphological sampling operators can be effectively applied to various image processing tasks, such as upscaling, denoising, or segmentation.

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Spectral Conditioning of Gram Matrices in Energy-Pooled Convolutional Classification Models

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We analyse the spectral conditioning of a supervised convolutional classification model with energy pooling and Softmax output. The energy feature of the c-th channel is the quadratic form $h_{i,c} = w_c^T G_i w_c$, where $G_i = X_i^T X_i / T$ is the Gram matrix of the zero-padded Toeplitz convolution matrix generated by the input signal x_i. We derive explicit gradient and block Hessian formulas showing that G_i governs the local geometry of the loss landscape through both the gradient directions $G_i w_c$ and the Gauss–Newton and residual-dependent Hessian terms.
The main result is a conditioning chain. The first link is the exact identity $\kappa_2(G_i) = \kappa_2(X_i)^2$, showing that the Gram construction quadratically amplifies ill-conditioning independently of the signal content and the optimisation algorithm. The second link connects $\kappa_2(G_i)$ to the power spectral density $\Phi_i(\omega)$ of the input signal: via the Wiener–Khinchin relation and Szegő's asymptotic spectral theory for Toeplitz matrices [1, 2],

$$\kappa_2(G_i) \approx \frac{\max_\omega \Phi_i(\omega)}{\min_\omega \Phi_i(\omega)}, \qquad M \ll T,$$

exact for circular convolution and asymptotically valid in the zero-padded Toeplitz case. We also study the average Gram matrix $\bar{G} = N^{-1} \sum_i G_i$ and establish the weighted bound $\kappa_2(\bar{G}) \leq \sum_i \theta_i \kappa_2(G_i)$, where $\theta_i = \lambda_{\min}(G_i) / \sum_j \lambda_{\min}(G_j)$. When the class spectra are complementary, the average symbol $\bar{\Phi} = N^{-1} \sum_i \Phi_i$ becomes flatter and $\kappa_2(\bar{G})$ can be reduced, giving a conditioning-based interpretation of balanced multi-class training.
The theoretical predictions are investigated on a real ECG arrhythmia database [3, 4] as an application case study. The rhythm class with the most spectrally concentrated profile exhibits the slowest gradient convergence, likely because the ill-conditioned Gram matrix produces flat gradient directions that may trap the optimiser in poor local minima. Mixing spectrally diverse classes reduces the effective condition number of the average Gram matrix, confirming the spectral complementarity effect.
These results show that, in energy-pooled convolutional models of this type, the root cause of optimisation difficulty is the spectral conditioning of the Gram matrix induced by the FIR parametrisation. To overcome this limitation, we are currently investigating IIR-based Gabor convolutions [5] as a structured, mathematically informed alternative. By replacing the CM free FIR coefficients with $2C$ interpretable spectral parameters $(\sigma_c, \omega_{0,c})$, the model searches directly over physically meaningful degrees of freedom rather than over an unconstrained time-domain space, with analytically controlled conditioning. This is the direction of our ongoing research.

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A nonlinear diffusion method for image processing

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This work presents a nonlinear diffusion method for image processing based on partial differential equations (PDEs).

Starting with the one-dimensional case, the proposed approach is progressively extended to two-dimensional images, three-dimensional volumetric data, and finally to multispectral and hyperspectral images.

The same nonlinear diffusion mechanism is employed throughout all these settings, providing a unified and flexible approach for processing imaging data of increasing complexity. The diffusion process preserves significant structures while smoothing homogeneous regions. The resulting numerical algorithms have low computational cost and require only one or a few iterations to produce meaningful results.

The proposed residual-based framework provides a unified approach to image enhancement, denoising, deblurring, structural analysis, and image segmentation, while maintaining a low computational cost suitable for large-scale imaging applications.

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Regularized spline approximation of planar offset curves

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In CNC machining and trajectory-processing pipelines, the manufacturing process typically starts from a CAD geometry, which is converted by CAM software into a tool path, often encoded as machine-readable code such as G-code. Before machining, the numerical control unit processes this path through a sequence of steps, including geometric data processing, contour smoothing, interpolation, and motion planning. Within this chain, the programmed path is often transformed into a smooth spline representation. As a consequence, planar curves arising in CNC applications are naturally modeled as spline curves, and subsequent geometric operations are expected to preserve this functional representation.

Offset computation is fundamental in toolpath generation and its accuracy directly determines how closely the machined contour matches the intended geometry. This operation does not, in general, preserve the spline representation: the offset of a spline curve is typically no longer a spline, and geometric singularities such as cusps or self-intersections may appear. Moreover, classical offsetting techniques usually do not account for the quality of the reconstruction of the original path. Consequently, offsetting an already approximated offset may fail to reproduce the original path with sufficient accuracy.

We present a family of regularization methods for the spline approximation of planar offset curves. Starting from a spline representation of a base trajectory, the proposed approaches compute an approximating offset spline by combining positional fitting with smoothness regularization. To improve bi-offset accuracy, we further incorporate Hermite-type conditions and parallelity-based constraints or penalties, which control the tangent directions of the approximating offset curve. The resulting framework preserves the spline representation and provides a robust approach that is well-suited for CNC toolpath generation, trajectory planning, and geometric modeling.

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Sharp error estimates for Lagrange interpolation and Kantorovich-type operators in weighted L_p-spaces

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In this talk, we present sharp direct and inverse approximation estimates for Lagrange interpolation polynomials and Kantorovich-type operators in weighted $L_p[-1,1]$ spaces. A well-known limitation in the theory of interpolation states that for small rates of convergence $\alpha \leq 1/p$, classical moduli of smoothness fail to provide matching direct and inverse estimates due to the insufficiency of integral metrics to determine pointwise values at the interpolation nodes. To overcome this difficulty, we introduce new special semi-discrete measures of smoothness based on weighted Steklov averaging operators, which simultaneously capture the global L_p-smoothness and average local smoothness of a function at the interpolation nodes. By using these measures, we establish fully matching direct and inverse approximation theorems for Lagrange interpolation polynomials, thereby providing complete two-sided characterizations of the rate of convergence for an arbitrary order $\alpha > 0$. To this end, we follow the approach developed in [1] and [2], where analogous problems were solved for functions on the torus and the real line, respectively. Furthermore, as a natural application of this approach, we discuss corresponding direct and inverse estimates, as well as strong converse inequalities, for Kantorovich-type operators constructed via Lagrange polynomials.

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Physics-Informed Feature Maps for Kernel-Based Approximation

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The problem of learning a functional relationship from finite data fits naturally within the theory of regularized approximation in reproducing kernel Hilbert spaces (RKHS) [2]. Given input-output data $\{(x_i, y_i)\}_{i=1}^n \subset \mathcal{X} \times \mathbb{R}$, with $\mathcal{X}$ a subspace of $\mathbb{R}^m$, one seeks $g \in \mathcal{H}_K$ minimizing

$$\min_{g \in \mathcal{H}_K} \left\{ \sum_{i=1}^{n} \ell(g(x_i), y_i) + \lambda \|g\|^2_{\mathcal{H}_K} \right\},$$

where ℓ is a loss function, $\lambda > 0$ a regularization parameter, and $\mathcal{H}_K$ the RKHS of a positive definite kernel K. The *representer theorem* [3] ensures the minimizer takes the form $g^*(x) = \sum_{i=1}^n \alpha_i K(x_i, x)$, reducing the problem to finite dimensions, with the kernel K encoding prior assumptions on the solution space.

In traditional applications, features are standardized and treated as abstract coordinates, neglecting structural information available in scientific problems—dimensional consistency, conservation laws, symmetries—thereby limiting interpretability and stability. We propose to construct *nonlinear, physics-informed feature maps* $\phi : \mathcal{X} \to \mathbb{R}^p$ respecting the physical nature of the problem. These maps induce a kernel

$$K_{\text{phys}}(x, x') = \langle \phi(x), \phi(x') \rangle,$$

embedding physical knowledge into the geometry of the hypothesis space. This also highlights a connection between kernel approximation and inverse problems [4]: choosing ϕ acts as *prior modeling*, so the approach can be read as a *physics-informed regularization strategy* where physical structure is embedded in the kernel itself, not only imposed through penalization.

Within this theoretic framework, symbolic regression methods such as PySR [5] can be interpreted as searching for explicit formulas within a restricted hypothesis space; however, they are computationally intensive, noise-sensitive, and focused on model identification rather than regularized approximation. The proposed approach instead uses physically meaningful feature maps to guide the construction of the hypothesis space, gaining tractability and extending to both regression and classification. This framework is validated on both synthetic and real data, covering two complementary scenarios. When the governing physics is known, a feature ranking procedure recovers the physically relevant terms and their coefficients accurately, even under significant noise, and compares favorably with PySR while being computationally more tractable. When the physics is unknown—as in solar flare forecasting—the same pipeline identifies the most predictive physics-informed features, providing interpretable candidates for the underlying dynamics and improving classification skill scores over standard feature-based approaches.

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II International Conference
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Fast computation of multi-dimensional volume potentials

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Many integral operators of mathematical physics are convolutions with singular kernel functions, for example with fundamental solutions of partial differential operators. Because of the singularity of the integrand, the numerical computation of those integrals by standard methods is an involved and time consuming task. The use of the quasi-interpolants introduced in the framework of *Approximate Approximations* ([1]) with adapted basis functions can be very advantageous. This approach, combined with separated representations, makes the method fast and effective also in multi-dimensions.

In this talk we show how this procedure can be applied, for example, to the cubature of the multi-dimensional harmonic potential and to the elastic and hydrodynamic potentials ([2]).

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Learning Under Environmental Shift: An Approximation-Theoretic Analysis of Climate Prediction

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Climate prediction models are often trained under environmental conditions that differ from those encountered at deployment, due to seasonal shifts, changes in circulation regimes, or longer-term climate trends. This work develops a theoretical framework, grounded in Barron-space approximation theory, to analyse and address this distributional mismatch [1,2]. Climate model responses are decomposed into a fast-varying component, capturing local weather dynamics, and a slowly varying modulation factor encoding large-scale environmental effects. This decomposition reflects the intuition that local patterns retain their structure across settings, while their intensity may shift across regions, seasons, or climate regimes. The main theoretical contribution is a clean error decomposition. Prediction error separates into an approximation term, arising from modelling the underlying climate dynamics, and an estimation term, arising from learning the environmental modulation. This result provides a principled basis for test-time adaptation, showing that even a small amount of data collected under new conditions can suffice to recalibrate an existing model. Furthermore, when the modulation factor varies smoothly, it can be learned efficiently from limited samples. This motivates a factorized architecture pairing a neural network for local climate variability with a lightweight module, such as a spline, a Fourier model, or a constrained network, for large-scale effects. Compared to treating the full process as a single function, this approach yields tighter control of prediction error and improved generalisation across varying environmental conditions.

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Evolution equations associated with positive linear operators

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We study a general class of Szász–Mirakjan–Durrmeyer type operators $S_{n,j}$ acting on exponentially bounded functions on $[0, \infty)$, with emphasis on their interpretation through an associated degenerate parabolic Cauchy problem. Using the evolution viewpoint together with differentiation properties of the operator family, we derive evolution equations satisfied by spatial derivatives of the generated solution and obtain propagation results for sign conditions on derivatives, leading to higher order shape-preserving properties. We also discuss explicit eigenstructures and separated-variable solutions, highlighting the role of classical special functions. The action of $S_{n,j}$ on exponential functions is employed to compute moments via differentiation with respect to the exponential parameter, yielding concrete formulas for low-order moments. A main focus is the interaction with $\{e_0, e_j\}$-convexity: we establish pointwise domination and monotonicity-in-time principles for $\{e_0, e_j\}$-convex data, compare $S_{n,j}$ with a discrete modification preserving the same test functions, and derive a Hermite–Hadamard type inequality adapted to $\{e_0, e_j\}$-convex functions, recovering the classical inequality as a particular case.

Keywords: General Durrmeyer type modification of Szász–Mirakjan operators; partial differential equation of parabolic type.

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CS-RBF Test Spaces for Patchless Meshfree VPINNs

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We propose an adaptive patchless Meshfree Variational-Physics-Informed Neural Network (MF-VPINN) for second-order elliptic Dirichlet problems. The aim is to retain the weak Petrov–Galerkin training mechanism of VPINNs and MF-VPINNs [1, 2] while removing the geometric layer still present in element-, subdomain-, or patch-based test spaces. The trial field is a standard neural network satisfying the boundary data through hard lifting. The test space is generated independently from anchor points: each anchor carries a Wendland C^2 compactly supported (CS) radial basis function (RBF) multiplied by the boundary factor, thereby defining a homogeneous test function. The CS-RBFs serve only as variational tests and residual localizers, not as neural activations or feature maps.

For each anchor, the weak residual is approximated by a quasi-Monte Carlo quadrature rule based on Sobol point clouds within its compact support; KD-tree radius queries identify the active samples. The resulting discretization uses no mesh generation, local triangulation, patch cutting, or geometric repair. Because adaptive refinement creates heterogeneous supports, the anchor residuals are scaled by $\sqrt{E(v_j)+\varepsilon}$, where $E(v_j)$ is the diffusion energy of the test function, and ε is a small positive regularization parameter to preserve floating-point accuracy definitions. This diagonal energy normalization balances coarse and fine anchors and improves loss conditioning.

Refinement is driven by a sampled strong-form residual indicator evaluated on Sobol candidate point clouds generated at each adaptive step, providing well-distributed residual-evaluation points over the computational domain Ω, combined with Dörfler marking, local child-anchor generation, and minimum-separation filtering.

Experiments on a singular Poisson benchmark and a perforated square with internal rectangular holes show that the CS-RBF anchor construction retains the adaptive behavior of patch-based MF-VPINNs while avoiding the patch layer. Anchors concentrate near singular or oscillatory regions, the indicator-free ablation stagnates, and energy normalization stabilizes nonuniform scales. In a reproduced patch-based baseline comparison, the patchless method gives steeper fitted relative H^1-error decay in five of six comparable indicator-driven strategy-CM settings and lower terminal relative H^1 errors in the corresponding runs.

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An inequality for Jacobi polynomials: a complement to Finite Increment Theorem

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Let $P = P_n^{(\alpha,\beta)}$ be the n-th degree Jacobi polynomial, which is orthogonal in $[-1,1]$ with respect to the weight function $(1-x)^\alpha(1+x)^\beta$, $\alpha,\beta > -1$. For parameters (α,β) satisfying either $\alpha \ge \beta \ge 1/2$ or $\alpha \ge 1/2$, $\beta = -1/2$, we prove the inequality

$$P(1) - P(x) \ge P'(x)\,(1-x), \quad x \in [0,1],$$

which may be viewed as a complement to Finite Increment Theorem for Jacobi polynomials.

This result was inspired by a refinement of two trigonometric inequalities due to M. S. Robertson (1945) and R. Askey and G. Gasper (1976), which also will be discussed in this talk.

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Simultaneous modular and norm approximation in Sobolev-Orlicz Spaces via semi-discrete operators

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In this talk, we consider a family of semi-discrete operators $\{S_w^{\varphi,\psi}\}_{w>0}$ generated by a discrete kernel φ and a continuous kernel ψ. Because of this mixed structure, these operators extend several classical approximation operators, including generalized sampling operators [1] and Kantorovich sampling operators. Moreover, they can also be viewed as quasi-interpolation operators playing a key role in harmonic analysis and shift-invariant spaces [5]; see also [2] for related regularization properties in L^p-spaces.

The aim of the talk is to study simultaneous approximation in Sobolev-Orlicz spaces $W^n L_\eta(\mathbb{R})$ in the sense of modular and Luxemburg norm convergence [6]. More precisely, we investigate the approximation of both functions and their weak derivatives, proving a modular convergence result of the form

$$\lim_{w\to+\infty} \rho_\eta\big(\lambda\big((S_w^{\varphi,\psi} f)^{(k)} - f^{(k)}\big)\big) = 0, \qquad k = 0, 1, \ldots, n,$$

for a suitable $\lambda > 0$ and a convex φ-function η.

The analysis is developed through two complementary approaches. The first one is based on density methods in Sobolev-Orlicz spaces [3]. We first establish convergence results on smooth compactly supported functions and then extend them to the whole space through a modular density result. A key role in this part is played by the Hardy-Littlewood maximal operator and its boundedness properties in Orlicz spaces [4,7].

The second approach is of direct type and relies on quantitative estimates for the rate of approximation. Despite requiring stronger assumptions on the kernels, this approach allows us to obtain convergence results under weaker assumptions on the φ-function and to work in a more general functional setting.

Finally, we discuss some recent developments concerning Voronovskaja-type formulae and related open problems in Sobolev-Orlicz spaces.

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Spectral Kernel Design in RKHS: A Tunable Meshless Collocation for Neutral Delay Equations

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Abstract. We introduce a novel family of positive definite radial kernels defined via the spectral density $1/(1 + (\sigma|\omega|)^{\alpha})$, where $\sigma > 0$ is a scale parameter and $\alpha > 0$ controls the smoothness. This construction yields a reproducing kernel Hilbert space (RKHS) with two continuous tuning parameters, offering significantly more flexibility than classical spline kernels. For even α, closed-form expressions are derived; in particular, for $\alpha = 4$ and $\alpha = 6$ the kernel takes the form of an exponential-trigonometric function, allowing efficient evaluation of the kernel and its derivatives. Using these kernels as trial functions in a collocation scheme, we develop a truly meshless numerical method for neutral functional-differential equations with proportional delay. The representer theorem guarantees that the approximate solution is a linear combination of kernel functions centered at the collocation points. The shape parameters α and σ enable continuous adaptation of the basis functions to the smoothness and localization required by the problem. An error analysis based on native space approximation provides theoretical justification for the convergence behavior of the method. Numerical experiments on several benchmark problems (including first-, second- and third-order neutral delay equations) confirm the theoretical rates and demonstrate high accuracy and computational efficiency.

Keywords: Reproducing kernel Hilbert space, meshless collocation, spectral kernel, shape parameter, neutral delay differential equations.

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Shape-preserving properties and localization results for max-product sampling Kantorovich operators

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This talk concerns the approximation properties for the family of max-product sampling Kantorovich operators studied in e.g. [4], within the framework of max-product approximation theory developed by Bede, Coroianu, and Gal [1]. These nonlinear operators are known to exhibit improved approximation behavior with respect to their classical linear counterparts.
We consider the operators

$$K_n^{\chi} f(x) = \frac{\bigvee_{k=0}^{n-1} \chi(nx-k) n \int_{\frac{k}{n}}^{\frac{k+1}{n}} f(u)du}{\bigvee_{k=0}^{n-1} \chi(nx-k)}, \qquad x \in [0,1],$$

where $f : [0,1] \to \mathbb{R}$ is locally integrable, and $\chi : \mathbb{R} \to \mathbb{R}$ is a suitable kernel function.
In [2], we investigated the shape-preserving properties of this class of operators, establishing a partial monotonicity preservation result. More precisely, under suitable assumptions on the kernel and the function f (see e.g. [3]), we proved that the monotonic behavior of the function is preserved by the corresponding operators. Moreover, we establish a localization property for this family of nonlinear operators. More precisely, if two functions coincide on a given interval, then, for sufficiently large values of n, the corresponding max-product sampling Kantorovich operators coincide on a subinterval sufficiently close to the original one. This highlights the local nature of the approximation process and shows that the reconstruction at a point depends only on values of the function in a neighborhood of that point.
The obtained localization and shape-preserving properties further emphasize the advantages of max-product sampling Kantorovich operators in the local approximation of functions, suggesting potential applications in signal reconstruction.

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Variably Scaled Kernels: A Kriging Perspective

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Classical Gaussian processes and Kriging models are commonly based on stationary kernels, whereby correlations between observations depend exclusively on the relative distance between scattered data. While this assumption ensures analytical tractability, it limits the ability of Gaussian processes to represent heterogeneous correlation structures. In this work, we investigate variably scaled kernels as an effective tool for constructing non-stationary Gaussian processes by explicitly modifying the correlation structure of the data. Through a scaling function, variably scaled kernels alter the correlations between data and enable the modeling of targets exhibiting abrupt changes or discontinuities. We analyse the resulting predictive uncertainty via the variably scaled kernel power function and clarify the relationship between variably scaled kernel based constructions and classical non-stationary kernels. Numerical experiments demonstrate that variably scaled kernels-based Gaussian processes yield improved reconstruction accuracy and provide uncertainty estimates that reflect the underlying structure of the data.

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Higher order approximation of continuous functions by the Goodman-Sharma-type operators

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Recently, Acu and Agrawal (2019) studied a family of modified Bernstein-Durrmeyer operators, so that for a special choice of certain parameters these operators lack the positivity but have a better than $O(n^{-1})$ order of approximation.

These results inspired I. Gadjev, P. Parvanov and R. Uluchev (2024, 2025, 2026) to introduce and explore new operators which can be explicitly defined using a unified approach. The operators under consideration are not positive and have $O(n^{-2})$ rate of approximation. In fact, these operators can be considered as Goodman-Sharma-type modifications of the Bernstein, Baskakov, and Meyer-König and Zeller operators, respectively.

By defining an appropriate K-functional, direct and strong reverse inequalities of type B, in the relevant terminology of Ditzian and Ivanov (1993), were proven.

The progress made and topics for further study are being discussed.

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A High-Dimensional Multifractal Fokker–Planck Model for Interacting Financial Markets: Classical and Quantum Perspectives

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The characterization of financial market dynamics through multifractal analysis has become an important tool for understanding scale-dependent fluctuations, long-range correlations, and crisis-induced structural changes [1]. In particular, the evolution of the singularity spectrum provides a compact description of the multifractal properties of a financial time series across different observation scales [2]. Existing approaches, however, are typically limited to the analysis of individual markets and do not explicitly account for interactions among multiple financial systems. In this work, we introduce a high-dimensional multifractal Fokker-Planck framework describing the joint evolution of singularity spectra associated with interacting financial markets. Starting from the transport equation describing the singularity spectrum of a market, we derive a multidimensional formulation in which the state of each market is represented by a local Hölder exponent. The resulting model incorporates scale-dependent drift terms, cross-market coupling effects, diffusion mechanisms associated with stochastic fluctuations, and external source terms representing exogenous shocks and crisis events. The proposed formulation naturally leads to a multidimensional probability density defined in the space of singularity exponents. The discretization through finite-difference schemes turns the problem into the solution of sparse linear systems whose dimension grows exponentially with the number of interacting markets. This motivates the investigation of advanced computational strategies, including quantum linear-system solvers, making the proposed framework a natural benchmark for comparing classical and quantum approaches to large-scale multifractal financial dynamics [3]. Particular attention is devoted to the computation of global multifractal observables and statistical indicators extracted from the joint singularity spectrum.

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POSTERS

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A Graph Approach to Music Score Compression via Connected Wedgelet Approximation and Tonnetz-Constrained Quantization

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Symbolic music scores can be naturally represented as graphs, where vertices encode musical events and edges describe harmonic and structural relationships. This representation enables score compression to be formulated as a graph approximation problem while preserving the intrinsic organization of the musical content. The proposed graph-based compression scheme recursively partitions the piano accompaniment into connected wedge regions, through an adaptive greedy algorithm operating in a six-dimensional Tonnetz embedding. Binary splits are driven by harmonic distance and constrained by shortest-path connectivity, ensuring that each wedge forms a connected subgraph representing a fragment of the score. Each wedge is approximated by its centroid in the Tonnetz space and subsequently mapped onto the discrete set of admissible pitch classes occurring in the original score through a Tonnetz-constrained quantization step. Consequently, the reconstructed score contains only original musical symbols, yielding a simplified, readable and directly playable score while preserving harmonic consistency. The proposed approach is evaluated on a corpus of symbolic scores from different composers. For each score, wedgelet approximations are computed at different compression ratios, and reconstruction accuracy is assessed by the Root Mean Square Error, allowing the compression/distortion trade-off to be quantified. Experimental results show that the proposed method achieves substantial score simplification while maintaining the essential harmonic structure and musical interpretability of the original compositions.

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Algebraic Semigroups, C_0-Semigroups and Generalized Convexity

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Let X be a compact convex subset of a Hausdorff locally convex real space and $C(X)$ the space of continuous real-valued functions on X. An *Altomare projection* $T \in B(C(X))$ is positive, its range contains the affine functions, and it satisfies $TKT = KT$ for every operator K in the algebraic semigroup $\mathcal{K} := \{K_{z,a} \mid z \in X, a \in [0,1]\}$, where $K_{z,a}f(x) := f(ax + (1-a)z)$. Such projections generate Bernstein–Schnabl operators B_n and, via the associated differential operator W_T, a C_0-semigroup whose infinitesimal generator A links subharmonicity to convexity-type properties of f.

We study operators $T \in B(C(X))$ possessing only the algebraic property $TKT = KT$, without assuming positivity, and characterise the resulting notions of T-convex and T-affine functions. We show $(\mathcal{K}, \circ)$ is a subsemigroup of $B(C(X))$ with identity $I_{C(X)}$, derive two De Casteljau-type algorithms computing $B_n f$, and prove that T-affine functions are exactly the fixed points of T and satisfy $B_n f = f$ for all $n \geq 1$, even when T is not positive.

In the positive case we revisit a 1999 conjecture of F. Altomare and I. Raşa relating the sign of $A(f)$ to weak T-convexity. We prove that if f is convex then the function $F(z,a) := Tf_{z,a}(z) - f(z)$ is increasing and convex in $a \in [0,1]$ for each $z \in X$, sharpening the known implication "convex $\Rightarrow$ generalized A-subharmonic". On the standard simplex $X \subset \mathbb{R}^2$ with the canonical projection T, we make the comparison between convexity and T-convexity explicit:

$$f''_{xx} \geq 0, \;\; f''_{yy} \geq 0, \;\; f''_{xx} + f''_{yy} - 2f''_{xy} \geq 0$$

characterises T-convexity (equivalently, axial convexity), strictly weaker than the classical Hessian condition $f''_{xx}f''_{yy} - (f''_{xy})^2 \geq 0$ for convexity.

These results connect the algebraic theory of semigroups of averaging operators with the analytic theory of C_0-semigroups and generalized convexity, and clarify the gap between the open Altomare–Raşa conjecture and the partial results currently available.

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Effective Localization and Forward Stability of Multinode Shepard Operators

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We study the effective localization and forward stability of multinode Shepard operators for scattered data approximation. Although these operators are globally supported, the product structure of their inverse-distance weights yields a quantitatively controlled decay of the normalized weights, resulting in an effectively local numerical action. Using a geometric mean distance associated with each multinode subset, we derive explicit decay estimates for the normalized weights and algebraic bounds for the contribution of distant subsets. These results provide a rigorous basis for truncated implementations with controlled error. We also derive weighted approximation and stability estimates in terms of local Lebesgue functions, and develop a finite-precision analysis based on logarithmic weight evaluation, log-sum-exp normalization, and backward stable local Vandermonde solves. Under the stated assumptions, the fully computed operator is shown to be first-order forward stable. Numerical experiments confirm the effective localization mechanism, the practical sharpness of the stability bounds, and the accuracy of the truncated approximations.

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On the interplay between power Lindley-type distributions and approximation processes

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The aim of this poster is to illustrate some properties of a family of probability density functions of exponential type and its connection with certain probabilistic positive linear operators.

Specifically, following the idea used for constructing the power Lindley distribution (see [2]), in [3] we introduced the following distribution

$$\begin{aligned} q_{\alpha,\sigma}(t,x) &= \frac{\alpha\beta(x)^{\frac{\sigma+1}{\alpha}+1}}{1+\beta(x)}\Gamma^{-1}\left(\frac{\sigma+1}{\alpha}\right)\left(1+\frac{\alpha}{\sigma+1}t^{\alpha}\right)t^{\sigma}\exp(-\beta(x)t^{\alpha}) \\ &= \frac{\beta(x)}{\beta(x)+1}p_{\alpha,\sigma}(t,x)+\frac{1}{\beta(x)+1}p_{\alpha,\sigma+\alpha}(t,x) \quad (t,\alpha,x>0,\sigma>-1), \end{aligned} \tag{1}$$

where β is a strictly positive continuous function on $(0,+\infty)$ and

$$p_{\alpha,\sigma}(t,x) = \alpha\beta(x)^{\frac{\sigma+1}{\alpha}}\Gamma^{-1}\left(\frac{\sigma+1}{\alpha}\right)t^{\sigma}\exp(-\beta(x)t^{\alpha}). \tag{2}$$

In particular, in [3] we computed some important related quantities of the distribution (1), such as the information potential, expected value, and variance. Moreover, we dealt with the possibility of determining some bounds for the information potential by means of certain integral operators of probabilistic-type introduced in [3] of the form

$$L_{\alpha,\sigma}(f)(x) = \int_0^{+\infty} q_{\alpha,\sigma}(t,x)f(t)\,dt$$

under suitable hypotheses on the parameters involved.

Note that density functions (2) were considered in [1] by Acu, Başcanbaz-Tunca and Rașa, where some interesting properties useful in Information Theoretic Learning were established.

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From Gaussian Kolmogorov–Arnold Networks to Adaptive Least-Squares Approximation: A Posteriori Error Estimation and Conditioning Limits

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Abstract

We study the adaptive approximation of linear elliptic boundary-value problems in which the trial space is a shared-basis Gaussian Kolmogorov–Arnold network (KAN), a structured radial-basis model. When the Gaussian centres are held fixed, the network depends linearly on its output coefficients: the discrete problem is exactly the least-squares finite element method associated with the boundary-value problem and is solved by a single linear least-squares system. Since the elliptic operator is an isomorphism from $H^2 \cap BC$ onto L^2, the least-squares residual is norm-equivalent to the H^2 error and is therefore a reliable and efficient a posteriori error estimator with effectivity index one. The residual drives an adaptive loop, Solve, Estimate, Mark (Dörfler), Refine, in which new Gaussian centres are inserted by nested enrichment. We prove reliability and efficiency of the estimator, an a priori approximation bound, a contraction property of the adaptive iteration, and a bound on the condition number in terms of the separation distance of the centres. Numerical experiments in one and two dimensions confirm the analysis: a measured effectivity index of one on smooth solutions, up to four times fewer degrees of freedom than uniform refinement on interior-layer solutions, and recovered convergence rates at algebraic and reentrant-corner singularities. A comparison with adaptive P_1 finite elements delimits the method's range: the C^∞ basis gives spectral-like accuracy per degree of freedom on smooth and sharp solutions, while the conditioning of the Gaussian basis imposes an accuracy floor that is reached, at isolated singularities, before the finite element error. We document this trade-off and characterise the regime in which the approach is advantageous.

Keywords: Kolmogorov–Arnold networks; radial basis functions; adaptive approximation; a posteriori error estimation; least-squares collocation; Dörfler marking; conditioning.

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Convexity, Positive Linear Operators and Stochastic Order Inequalities

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Abstract

We investigate inequalities generated by positive linear operators acting on convex functions and their interpretation in terms of convex stochastic ordering.
Using an operator-based approach, we establish a unified framework that includes several classical approximation processes such as Bernstein, Baskakov, Szász-Mirakjan, Beta, Gamma, Post-Widder and related operators.
New inequalities are obtained together with monotonicity properties and shape-preserving results.
Besides the direct problem, where convexity implies operator inequalities, we also address the inverse problem of determining whether inequalities between suitable positive linear operators characterize convex functions.
Several examples and open questions are presented.

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Uniform and non-uniform spline quasi-interpolation for the numerical solution of second-kind integral equations with weakly singular kernels

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In the last years, spline quasi-interpolation (QI) has been successfully employed for the numerical solution of integral equations of the second kind with various types of kernels (see e.g. [1]), provided that the exact solution of the considered integral problem has some continuous derivatives all over the domain. However, in case of weakly singular kernels, in general, the solution can be differentiable many times in the interior of the domain, while it can be only continuous at the boundary, negatively affecting the convergence order of the classical numerical methods based on QI [2]. Therefore, the aim of this contribution is to propose two product-integration schemes based on spline QI that preserve the high convergence order also in presence of solutions not differentiable at the extrema of the domain: the first scheme is based on uniform spline QI applied to a regularized version of the integral problem, while the second one is constructed employing non-uniform QI on graded meshes.

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Spike Sorting and Spike-based Clustering of HD-MEA Brain Organoid Recordings

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High-density Microelectrode Arrays (HD-MEAs) represent a major technological advancement in neurophysiology, enabling the non-invasive recording of thousands of extracellular signals from neuronal populations and providing a powerful tool for investigating complex network dynamics. However, the high dimensionality of the acquired data requires efficient computational methods for analysis and interpretation. In this work, we propose a framework for clustering neural organoid activity under different experimental conditions through dimensionality reduction of HD-MEA recordings. The proposed approach combines spike detection and spike sorting techniques to extract compact representations of neural activity. First, active electrodes, defined as those exhibiting a mean firing rate greater than 0.25 *spikes/s*, are identified, and p descriptive metrics related to spike and burst activity are computed for each electrode. In parallel, a template-matching spike sorting algorithm is employed to identify the n neuronal units present in each recording, and the same set of features is extracted for each detected unit. Consequently, the original recording, represented as a matrix $X \in \mathbb{R}^{s \times T}$, where T denotes the number of temporal samples and s the number of recorded signals, is transformed into two lower-dimensional representations: a matrix $X_{active} \in \mathbb{R}^{a \times p}$ associated with the $a \leq s$ active electrodes and a feature-based representation of the identified neuronal units as n matrices $X_i \in \mathbb{R}^{a \times p},\ i = 1, \cdots, n$. A similarity measure derived from the Earth Mover's Distance is then introduced to quantify distances between recordings and support clustering across experimental conditions. The proposed methods were evaluated on a dataset obtained from a two-dimensional culture seeded with 1,000 cells and maintained in vitro for 83 days, with recordings acquired at approximately 7–10 day intervals. Experimental results indicate that the spike-sorting-based representation is more effective than in discriminating different stages of in vitro cellular development, highlighting the potential of neuronal-unit-level features for characterizing the temporal evolution of neural cultures.

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Privacy Factor and Relative Fisher Geometry for Parametric Stochastic Processes

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We develop an information-geometric theory of privacy for parametric stochastic processes observed in real time. A physical system driven by a parameter trajectory $\theta(t)$ is measured through a noisy channel; an adversary accumulates observations and attempts to distinguish the true trajectory from a nominal reference. The *absolute privacy factor* is defined as $\bar{\Pi}_{\text{abs}} := \exp(-\bar{D}(P\|Q)) \in (0,1]$, where $\bar{D}(P\|Q) = \limsup_{T\to\infty} T^{-1} D_{KL}(P^{(T)}\|Q^{(T)})$ is the KL rate of the true law P against the reference law Q. The value $\bar{\Pi}_{\text{abs}} = 1$ corresponds to maximal privacy (zero divergence rate), while $\bar{\Pi}_{\text{abs}} \to 0$ means the adversary can identify the trajectory asymptotically.

The space-time parameter manifold $\hat{\Theta} := [0,T] \times \Theta$ carries a *relative Fisher metric* $\tilde{g}_{ab}(\xi) = (\partial_a \tilde{\mu})^\top \Sigma^{-1} (\partial_b \tilde{\mu})$, where $\tilde{\mu} = \mu(\xi) - \mu^q$ is the deviation of the observation mean from the reference. The associated *relative informational energy* $\tilde{E}[\gamma] = \frac{1}{2}\int_a^b \tilde{g}_{ab}\dot{\gamma}^a\dot{\gamma}^b\,ds$ measures the squared velocity of the deviation curve weighted by the inverse sensor covariance. An asymptotic upper bound on the KL divergence in terms of $\tilde{E}[\gamma]$ then yields a lower bound on $\bar{\Pi}_{\text{abs}}$.

Stationary curves of the Rayleigh quotient $\mathcal{R}[\eta] := \tilde{E}[\eta]/K[\eta]$, where $K[\eta] := \int \|\dot{\eta}\|^2\,ds$ is the Euclidean kinetic energy, satisfy a variational eigenvalue problem $\mathcal{E}(\eta, v) = 2\lambda\,\mathcal{K}(\eta, v)$ for all test functions v. Under standard coercivity assumptions the spectrum is real, discrete and bounded below, and the first eigenvalue λ_1 characterises the optimal privacy-efficiency trade-off:

Theorem 1. *Among all parametric trajectories with fixed kinetic budget $K[\eta] = K_0$, the minimum informational leakage equals $\lambda_1 K_0$, attained by the first eigenfunction $\eta^{(1)}$. Consequently, $\bar{\Pi}_{\text{abs}} \geq \exp\big(-\frac{(b-a)^2}{2\Delta t}\lambda_1 K_0\big)$.*

In the *linear Gaussian* case—$\dot{x} = A(t)x + B(t)\theta$, $y = Cx + \nu$, $\nu \sim \mathcal{N}(0, \Sigma_0)$—the bilinear forms $\mathcal{E}$ and $\mathcal{K}$ are explicit. Using the adapted sine basis $\phi_{j,\alpha}(s) = \sin\big(\frac{(2j-1)\pi}{2(b-a)}(s-a)\big)\hat{e}_\alpha$, which satisfies the Dirichlet condition at $s = a$ and the natural transversality condition at $s = b$, the Galerkin method reduces the infinite-dimensional problem to a finite generalised eigenproblem $\mathbf{M}_E\mathbf{c} = \lambda\mathbf{M}_K\mathbf{c}$ with diagonal mass matrix $\mathbf{M}_K$. The approximations $\lambda_k^{(N)}$ converge *monotonically from above* to the true eigenvalues λ_k, with error $\lambda_k^{(N)} - \lambda_k = O(N^{-2r})$ where r is the Sobolev regularity of the k-th eigenfunction.

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Monte Carlo option pricing with Quantum Random Numbers: A Statistical Quality Assessment

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We compare NumPy's PCG64 pseudo-random number generator with six quantum random number streams produced by a superconducting nanostrip photon-number detector. All sources are passed through the same 32-bit inverse-CDF pipeline and used in a Monte Carlo Black-Scholes pricer for a European call on the geometric average of five NASDAQ equities, for which an exact benchmark is available.

PRNG, Method 1, and Method 2 with $k \in \{2, 4, 8\}$ pass the statistical quality tests and are empirically indistinguishable in pricing accuracy, recovering the expected $\mathcal{O}(n^{-1/2})$ Monte Carlo convergence rate. In contrast, Method 2 with $k = 16$ and $k = 32$ shows significant bit-frequency bias, fails standard uniformity tests and does not converge: its RMSE remains dominated by a persistent bias even as the sample size increases.

A controlled Poisson bias sweep confirms that the deterioration in Monte Carlo performance occurs at the same threshold detected by the NIST frequency test. These results show that the quantum origin of the random bits is not sufficient to guarantee numerical reliability, the statistical quality of the extraction procedure is the decisive factor for Monte Carlo accuracy.

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